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Minimax Approach for Using the Qualitative Preferences in the Multicriteria Evaluation

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Abstract—Decision support systems in analytical systems based on the use of big data involve the formation of integral assessments of population objects using all parameters or some subset of them. The article discusses the problem of obtaining a multi-objective (multi-parameter) assessment of objects and an approach that involves the use of importance weights in the presence of high-quality and, possibly, incomplete information about the relative importance of certain criteria. The fundamental principle of various quantitative assessments of the mutual preference of private particular criteria for various objects in the population is considered while maintaining the system of preferences of the entire set of objects. The approach used assumes that the decision maker formulates qualitative information about the relative preference of certain criteria in the form of a not necessarily complete preference graph. For each object, weighting coefficients are calculated automatically according to the principle of a guaranteed result by solving an optimization problem using generalized logical criteria of maximum risk and maximum caution. For special cases of preference systems, analytical relationships and algorithms for calculating weight coefficients are given. This technique ensures the use of additional qualitative information about the preferences of certain criteria, obtaining numerical values of significance weighting coefficients and solving the problem of multicriteria assessment based on the principle of a guaranteed result.

Keywords: multicriteria optimization, multicriteria estimation, interactive procedure, weighting coefficients, qualitative preference information

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1. INTRODUCTION

Problems of multicriteria assessment are widespread in various fields of activity, and, consequently, in various subject areas. In this case, the assessment task is understood as the situation of the need to obtain an integral assessment of the objects in the population for the subsequent final choice of the decision maker. The objective complexity of such problems lies in the impossibility of achieving the best values for all criteria at the same time, so we are talking about choosing a compromise solution, that is, a solution that cannot be improved by any of the criteria without worsening the other criteria. One of the most common approaches to solving such problems is to generate a numerical value estimate for each feasible solution and therefore solve a single-criteria optimization problem or, in the case of a discrete set of feasible solutions, a selection problem [1].

There are approaches to the problem of multi-criteria choice that take into account qualitative information about the preferences of decision makers [2]. The main disadvantage of these methods is the requirement for the decision maker to introduce a sufficiently large number of additional parameters (exact numerical values), the absence (at least partial) of which makes the applicability of these methods impossible. The described technique involves the use of exactly the amount of information that the decision maker is ready to provide (in a specific case, the absence of such information at all) [4–6].

One of the most common methods for solving multicriteria (multiobjective) problems is the use of a generalized optimality criterion, which includes importance weights that reflect the decision maker's idea of the relative importance of certain optimality criteria [2]. The importance of criteria is understood here in the sense of the axiomatic theory of importance, which allows us to assume that if there is additional information of the form the ith criterion is more preferable than the jth criterion ($Q_i \succ Q_j$), then for weighted coefficients, the ratio $w_i > w_j$ should be true. In this case, of course, the decision maker requires exact numerical values of the weighting coefficients.

The weights may be assigned by the decision maker in various ways [3]. All known methods of individual examination require from the decision maker or expert complete and accurate answers (most often with numerical estimates) to the questions that the chosen method asks. The described approach allows the use of incomplete and/or qualitative information about the preferences of particular criteria.

2. MULTICRITERIA ESTIMATION PROBLEM STATEMENT

In general, the problem of multi-criteria choice can be formulated as follows.

The decision maker (DM) has formed a set of a number of feasible alternatives, from which DM needs to make a single and final choice. The choice is made as selection of a certain subset of alternatives (in a particular case, one alternative) such that the decision maker considers them the best and equivalent to each other in terms of the best quality.

Also in practice, the problem of multicriteria (also multiobjective) evaluation of options is considered and solved, as a result of which the decision maker seeks to order the options by preference for further analysis. Obviously, this task (estimation) is a generalization of the previous one (choice) and does not contradict practice.

Traditionally, the following components of the decision-making model are considered:

— initial set of options (solutions, alternatives) from which the choice is made;

— the principle of optimality, on the basis of which the best option (or best options) is selected;

— decision maker (DM), who determines the process of finding a solution, as well as experts and consultants who assist him in this.

Without limiting the generality of the mathematical formulation, we will assume that the set of feasible solutions is a discrete finite set of alternatives, each of which is denoted by a number:

$$D = \{1, \dots, m\}. \tag{1}$$

The choice task is to choose the variant $x^* \in D = \{1, 2, \dots, m\}$, which is the best (optimal) from the decision maker's point of view.

Since there is no optimal solution in most cases of a multi-criteria choice problem, the term "rational decision" has become established in the literature, which means the presence of rational reasons understandable to other people that led to the choice of this solution from a set of acceptable ones.

Many approaches to solving problems of multi-criteria choice involve the use of an estimation problem. The task of evaluation is to determine for each alternative a numerical value that characterizes the quality (utility, efficiency, etc.) of this option in terms of the subject area and the relative importance of particular criteria in accordance with the individual opinion of the decision maker. At the same time, for comparability of alternatives, this generalized characteristic should have a direction and scale.

In the future, we will assume that the measurement scales of particular optimality criteria are defined and the numerical value of the assessment by option can be obtained for each option and each criterion.

We will also assume that all partial criteria are reduced to a dimensionless form and to a single measurement scale $[\alpha, \beta]$, while $0 \leqslant \alpha < \beta$.

The general idea of evaluation is to construct a scalar function $F(Q(x))$, which has the property of ordering the original options according to their preference. The procedure for constructing such a function is called folding or combining the vector optimality criterion (as an option, folding or combining particular criteria):

$$\min_{x \in D} Q_1(x), \quad \min_{x \in D} Q_2(x), \quad \cdots, \quad \min_{x \in D} Q_n(x), \tag{2}$$

subject to

$$D = \{1, 2, \ldots, m\} \tag{3}$$

reduces to a one-criterion (scalar) problem

$$\min_{x \in D} \{F(Q_1(x), \ldots, Q_n(x))\}. \tag{4}$$

The function F is called the generalized criterion for the optimality of the problem (2). To take into account the relative importance of particular criteria, weights of the relative importance $w = (w_1, \ldots, w_n)$ are used. Various functions can be used as a generalized criterion, particularly generalized logical criteria of optimality:

"maximal caution" criterion:

$$F_{L_{\min}}(w, Q_1(x), \ldots, Q_n(x)) = \min_{1 \leqslant i \leqslant n} (w_i Q_i(x)), \tag{5}$$

"maximal risk" criterion:

$$F_{L_{\max}}(w, Q_1(x), \ldots, Q_n(x)) = \max_{1 \leqslant i \leqslant n} (w_i Q_i(x)). \tag{6}$$

Where the weights $w_j,\ j = 1, \ldots, n$ reflect DM's view of the importance of private optimality criteria. The importance of criteria is understood here in the sense of the axiomatic theory of importance, which allows us to assume that if additional information of the form ith criterion is no less important than the jth criterion ($Q_i \succeq Q_j$), then the following relation is true for the weight coefficients and:

$$Q_i \succeq Q_j \Leftrightarrow w_i \geqslant w_j. \tag{7}$$

Again, to solve the original selection problem, the exact assignment of weighting coefficients of importance in numerical form is required. In this case, it is usually assumed that the weighting coefficients of importance belong to the region of admissible values of the following form (hereinafter we will refer to it as the region of admissible values of the weighting coefficients):

$$D_w^0 = \left\{ w \in R^n |\ w_i \geqslant 0, \quad i = 1, 2, \ldots, n; \quad \sum_{i=1}^{n} w_i = 1 \right\}. \tag{8}$$

It is known that the choice of weight coefficients of the importance of particular criteria in strict accordance with can lead to a certain problem: if for some particular criterion Q_i the value of the corresponding weight coefficient of importance is zero (which does not contradict the construction of the region D_w), then this criterion will essentially be struck out of consideration and it makes no difference what value it will have. Mathematically, this will lead to a weak efficiency of the solutions obtained (proof in [Podinovsky, Nogin]).

In addition, it is known that the sum of the importance weights can be any positive number (for example, equal to 100, if it is convenient to work in percentage terminology).

Therefore, we finally come to the following formulation of the range of feasible values of weight coefficients of importance:

$$D_w^1 = \left\{ w \in R^n |\ w_i \geqslant w_0 > 0, \quad i = 1, 2, \ldots, n; \ \ \sum_{i=1}^{n} w_i = R \right\}. \tag{9}$$

To solve the original selection problem, the decision maker needs to assign the exact values of the importance weights in numerical form from the range of their feasible values.

Approaches to the definition and methods for assigning weighting coefficients are widely described in the literature. Among them are the following [5]:

— ordering criteria by importance;
— determining the ratio of weight coefficients, while the decision maker specifies the ratio $w_i w_j$ in numerical form;
— construction of tables based on pairwise comparison of criteria by importance;
— method for determining weights using a set of sequential comparisons (Churchman-Akoff method);
— methods that use information about the quality of the optimal values of particular criteria;
— game-theoretic methods for assigning weight coefficients and others.

These methods have a number of significant drawbacks in their practical application.

(1) The statement (7) implies a “transition” from qualitative information ($Q_i \succeq Q_j$) to quantitative information ($w_i \geqslant w_j$). Obviously, this is a big uncertainty, which will eventually lead to different solutions.
(2) All of the listed methods for assigning numerical values of weight coefficients of importance are sequential algorithms that require accurate quantitative information from the decision maker at each step of this algorithm. As soon as the decision maker refuses to answer the next question, the entire algorithm terminates and the numerical values of w are not calculated.

3. ANALYSIS AND USE OF QUALITATIVE INFORMATION ON THE RELATIVE IMPORTANCE OF PARTICULAR OPTIMALITY CRITERIA – MINIMAX APPROACH

In a number of works [4–6], the main idea and concept of this approach are formulated, which are as follows.

(1) It is assumed that when using generalized optimality criteria with weighting coefficients of importance, their values can be different for each feasible solution of the domain D.
(2) The decision maker can formulate preferences on the set of particular criteria in the form of a set of relations, not necessarily for all possible pairs of particular criteria.
(3) Importance weight values are calculated automatically based on the guaranteed result principle. That is, for each feasible solution, such a set $w \in D_w$ is determined that provides the “worst” value of the generalized criterion for all possible values of the weight coefficients.

Let us consider in more detail a particular, but often occurring case, in which the decision maker establishes mutual preference for some L pairs of particular criteria (not necessarily for all C_n^2 possible pairs) preference of the ith particular criterion over the jth one on the entire set D of feasible solutions:

$$e_l = \{Q_i \succeq Q_j\}, \ l = 1, 2, \ldots, L \leqslant \frac{n(n-1)}{2}. \tag{10}$$

The information (10) is qualitative, as it follows from it that the criterion i is more important than the criterion j, but it is impossible to say how much or how many times. Then, taking into account the relation (10), the range of acceptable values of the weight coefficients of importance D_w, defined as (8), narrows according to the relations introduced by the decision maker:

$$D_w^2 = \left\{ w \in R^n \,\middle|\, \begin{array}{l} w_i \geqslant w_0 > 0,\ i = 1, 2, \ldots, n; \\ \sum_{i=1}^{n} w_i = R; \\ \langle w_i \geqslant w_j \rangle_{e_l},\ l = 1, 2, \ldots, L \end{array} \right\}. \tag{11}$$

In some cases, the decision maker has the ability to clarify information about the relationship of weight coefficients w_i and w_j associated with a binary relation $\{Q_i \succeq Q_j\}$ using the parameter $g_l \geqslant 1$:

$$D_w^3 = \left\{ w \in R^n \,\middle|\, \begin{array}{l} w_i \geqslant w_0 > 0,\ i = 1, 2, \ldots, n; \\ \sum_{i=1}^{n} w_i = R; \\ \langle w_i \geqslant g_l w_j \rangle_{e_l},\ l = 1, 2, \ldots, L \end{array} \right\}. \tag{12}$$

If the ares of feasible values of the weight coefficients (11) and (12) are not empty, then the qualitative information is consistent.

In several works [4, 6], an approach was proposed and developed in which the weight coefficients of the importance of particular criteria are uncontrollable factors, and their values can be different for different alternatives.

Sometimes, in the decision making process, it is reasonable to take into account the dependence of the weight coefficients on the value of the particular criteria at each point of the domain of feasible solutions. Assuming that the decision maker can not accurately determine the numerical values of the weight coefficients w, they can be considered as uncontrolled factors and, applying the principle of guaranteed result, proceed to the following decision-making model:

$$x^* = \arg\min_{x \in D} \max_{w \in D_w} F(q(x), w) \tag{13}$$

subject to

$$D = \{1, 2, \ldots, N_D\}, \tag{14}$$

$$D_w^3 = \left\{ w \in R^n \,\middle|\, \begin{array}{l} w_i \geqslant w_0 > 0,\ i = 1, 2, \ldots, n; \\ \sum_{i=1}^{n} w_i = R; \\ \langle w_i \geqslant g_l w_j \rangle_{e_l},\ l = 1, 2, \ldots, L \end{array} \right\}. \tag{15}$$

4. QUALITY INFORMATION GRAPH

Qualitative information about the expert's preferences can be represented as a directed graph $G(V, E)$, where V is the set of vertices corresponding to particular criteria, E is the set of edges connecting the ith vertex with the jth vertex if and only if the relation $(Q_i \succeq Q_j)$ is completed.

In what follows, we will assume that information messages (7) satisfy the transitivity condition, that is, there are no closed cycles in the graph $G(V, E)$.

Let us divide the set I of graph vertices into tiers as follows:
— the first tier ($s = 1$) includes all vertices that do not include any arc;
— to the second tier ($s = 2$) those and only those vertices that include arcs from the vertices of the first layer;
— to the third tier ($s = 3$) those and only those vertices that include arcs from the vertices of the first and second layers, etc.

In this case, the vertices of the last tier ($s = S \leqslant N$) will not have a single outgoing arc.

In the special case of the absence of qualitative information (7), the graph $G(V,E)$ will be a collection of N isolated vertices. And in the particular case of linear ordering of particular criteria, the graph $G(V,E)$ will have the number of layers equal to the number of vertices ($S = n$).

As an example, consider a preference graph (Fig. 1) containing six vertices (criteria) and additional qualitative information introduced by the decision maker: $e_1 = \{Q_6 \succeq Q_4\}$, $e_2 = \{Q_6 \succeq Q_3\}$, $e_3 = \{Q_4 \succeq Q_1\}$, $e_4 = \{Q_3 \succeq Q_1\}$, $e_5 = \{Q_3 \succeq Q_2\}$, $e_6 = \{Q_1 \succeq Q_5\}$.

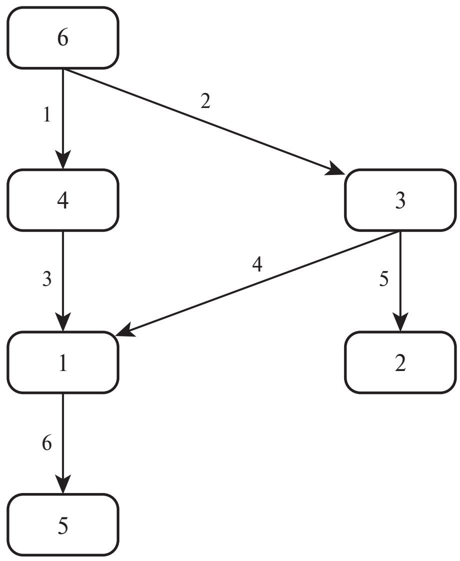

Fig. 1. Example of the preference graph.

For each vertex of the graph $v_i \in \{1, 2, \ldots, n\}$ corresponding to the particular criterion Q_i, we introduce the following notation: V_i is the set of vertices of the graph $G(V,E)$ from which there is a path to vertex i, including itself; n_i is the cardinality of the set V_i.

Let p be an arbitrary path in the graph $G(V,E)$: $p = \{l_1, \ldots, l_n(p)\}$, where $l_i \in E$ is an arc included in the path p; $n(p)$ is the number of arcs in the path p. Denote by P_i^k the set of all paths from vertex i to vertex k. Then we introduce the quantities $\bar{g}_k^i$ and $\bar{\bar{g}}_i$ as follows:

$$\bar{g}_k^i = \begin{cases} \max\limits_{p \in P_i^k} \prod\limits_{l \in p} g_l, & P_i^k \neq \emptyset \\ 1, & P_i^k = \emptyset \end{cases} \tag{16}$$

for each $i, k \in E$;

$$\bar{\bar{g}}_i = \max_{k \in E, k \neq i} \bar{g}_k^i. \tag{17}$$

Obviously, $\bar{\bar{g}}_i$ is a generalized estimate of the relative importance of a particular criterion i under the conditions of qualitative information (10). An example of calculating these quantities for the graph shown in Fig. 1 is below. For all arcs of the graph, let us assign refinement coefficients g_l, $l = 1, \ldots, 6$ (Table 1).

Table 1. The example of refinement coefficients

Number of arc l	Coefficient g_l
1	1.2rule0pt3.4mm
2	1.3
3	1.0
4	1.5
5	1.3
6	2.0

Calculation of values (16) and (17) for all vertices of the graph is given in Table 2.

Table 2. The example of vertex characteristic calculation

Vertex i	Values of $\bar{g}_i^k (P_i^k \neq \emptyset)$	$\bar{\bar{g}}_i$
1	$\bar{g}_1^5 = 2.0$	2.0
2	–	1.0
3	$\bar{g}_3^5 = 2.0 \times 1.5 = 3.0; \bar{g}_3^2 = 1.3$	3.0
4	$\bar{g}_4^5 = 1.5 \times 1.0 = 1.5$	1.5
5	–	1.0
6	$\bar{g}_6^5 = \max\{1.2 \times 1.0 \times 2.0; 1.3 \times 1.5 \times 2.0\} = 3.9; \bar{g}_6^2 = 2.0 \times 1.3 = 2.6$	3.9

Algorithm 1

Require: $s = S; \bar{g}_i^k = 0, i, k \in \{1, \ldots, n\}$.

1: For all vertices i from tier s let $\bar{\bar{g}}_i = 1$.

2: $s \leftarrow s - 1$. If $s = 0$ (that is, all tiers are passed), then go to step 4, otherwise go to step 3.

3: For each vertex k belonging to layer s, we perform the following actions. Let J_k be the set of vertices in the layer $(s+1)$ that have an arc from vertex k. Then, for each vertex $j \in J_k$, we put:

$$\bar{g}_i^k = g_l, \text{where} \quad l \quad \text{is number of vertex } (k, j) \quad \text{in the graph} \quad G(V, E);$$

$$\bar{g}_k^i = \max_{j \in J_k} \left\{ \bar{g}_k^j; \bar{g}_j^i \right\}, i \neq j; \;\; \bar{\bar{g}}_k = \max_{j \in J_k} \left\{ \bar{g}_k^j; \bar{\bar{g}}_k \right\}$$

and go to step 2.

4: For all $\bar{g}_i^k$, for which $\bar{\bar{g}}_k = 0, k, i \in \{1, \ldots, n\}$ assign $\bar{g}_i^k = 1$.

5. WEIGHTING COEFFICIENTS CALCULATION FOR MAXIMAL CAUTION CRITERIA

5.1. Common Case

Consider the solution to the problem of calculating weight coefficients of importance $w \in D_w^4$ using the generalized logical criterion "maximum caution":

$$x^* = \arg \min_{x \in D} \max_{w \in D_w} \max_{1 \leqslant i \leqslant n} F(q(x), w), \tag{18}$$

$$D_w^1 = \left\{ w \in R^n \;\middle|\; \begin{array}{l} w_i \geqslant w_0 \geqslant 0, \; i = 1, 2, \ldots, n; \\ \sum_{i=1}^{n} w_i = R; \\ \langle w_i \geqslant g_l w_j \rangle_{e_l}, \; l = 1, 2, \ldots, L \end{array} \right\}. \tag{19}$$

For fixed values of the variable parameters $x \in D$, the values of the vector criterion $q(x)$ are definite.

Applying a linear model for determining mixed strategies in a zero sum matrix game [14], in the general case, the solution to problem (18)–(19) can be obtained by solving the following linear optimization problem:

$$\max v \tag{20}$$

subject to

$$D = \left\{ w \in R^n \,\middle|\, \begin{array}{l} w_i \geqslant w_0 \geqslant 0,\ i = 1, 2, \ldots, n; \\ \sum\limits_{i=1}^{n} w_i = R; \\ \langle w_i \geqslant g_l w_j \rangle_{e_l},\ l = 1, \ldots, L \\ w_i q_i(x) \geqslant v,\ i = 1, \ldots, n \end{array} \right\}. \tag{21}$$

Consider an example of calculating weight coefficients. Let us assume that the values of partial optimality criteria for one feasible solution, normalized to the interval $[0, 1]$, are given in Table 3. In this table calculated weighting coefficients are given as well.

Table 3. The example of criteria values of the feasible solution and calculated weighting coefficients

Criteria	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6
Criteria values q_i	0.1	1.0	0.5	0.3	0.9	0.3
Weighting coefficients w_i	0.1766	0.1767	0.2650	0.1767	0.0196	0.3445

The above solution to the problem of calculating the weight coefficients of importance of particular criteria was obtained using the LPSolv software package [15]. The source code in LP format to solve the problem is given below.

Listing 1. Source LP code

```
/* Objective function */
max: v;

/* Variable bounds */
w1 + w2 + w3 + w4 + w5 + w6 = 1;

w1 >= 0.01;
w2 >= 0.01;
w3 >= 0.01;
w4 >= 0.01;
w5 >= 0.01;
w6 >= 0.01;

w6 >= 1.2 * w4;
w6 >= 1.3 * w3;
w4 >= 1.0 * w1;
w3 >= 1.5 * w1;
w3 >= 1.3 * w2;
w3 >= 2.0 * w5;
```

```
0.1 w1 >= v;
1.0 w2 >= v;
0.5 w3 >= v;
0.3 w4 >= v;
0.9 w5 >= v;
0.3 w6 >= v;
```

The above solution was obtained by solving a linear optimization problem using the simplex method. For special cases of qualitative information about preferences and, therefore, the range of acceptable values of importance weight coefficients, the calculation of coefficient values can be carried out by faster algorithms.

5.2. Calculation in Particular Cases

Consider the solution to the problem of calculating weight coefficients of importance $w \in D_w^2$ (11) using the generalized logical criterion "maximum caution":

$$x^* = \arg\min_{x \in D} \max_{w \in D_w} F(q(x), w), \tag{22}$$

$$D_w^1 = \left\{ w \in R^n \;\middle|\; \begin{array}{l} w_i \geqslant w_0 \geqslant 0,\ i = 1, 2, \ldots, n; \\ \sum\limits_{i=1}^{n} w_i = R; \\ \langle w_i \geqslant w_j \rangle_{e_l},\ l = 1, 2, \ldots, L \end{array} \right\}. \tag{23}$$

To calculate the values of the weight coefficients of importance w in the case of the condition of their non-negativity, that is, for the case $w_0 = 0$, the Algorithm 2 can be applied.

Algorithm 2

1: The graph $G(V, E)$ is divided to tiers.
2: Assigning an initial value $w_j' \leftarrow 0$ to each vertex j of the graph $G(V, E)$, $j = 1, \ldots, n$.
3: Consider the lowest layer as the first layer: $s \leftarrow S$.
4: $w_j' \leftarrow \max\left\{w_j', R/q_j(x)\right\}$ to all vertices j of tier s.
5: Adjustment of w_j' values for vertices of higher layers:

$$w_j' = \max\left\{w_j', \max_{k \in V_j} w_k'\right\},$$

V_j is the set of vertices, which have path to vertex j.
6: Let $s = s - 1$. If $s = 0$ i.e. all tiers have been passed, go to step 7. Otherwise repeat from the step 4.
7: Calculate final values of weighting coefficients:

$$w_j^* = \frac{R \times w_j'}{\sum\limits_{i=1}^{n} w_i'}$$

Consider a generalization of the extrema problem (22)–(23) for the case $w_0 \geqslant 0$. To solve this problem, the Algorithm 3 can be applied.

The proof of the correctness of the above algorithms is contained in [5].

Algorithm 3

Require: $V' = V = \{1, \ldots, n\}; R' = R; E' = E$.

1: Apply Algorithm 2 to graph $G(V, E)$ with $R = R'$ and get values w'_j.
2: If for all $j \in V'$ it is true $w'_j \geqslant w_0$ then assign $w^*_j = w'_j, j \in V'$ and the task has been solved. Otherwise go to the step 4.
3: For all vertex $m \in V'$, for which $w'_m < w_0$, make the following:
   — assign $w'_m = w_0; R' = R' - w_0$;
   — exclude the vertex m from consideration, for this assign $V' = V' \setminus m$ and from the set E' we exclude the arcs incident to the vertex m, if any.
4: Repeat from step 1.

6. WEIGHTING COEFFICIENTS CALCULATION FOR MAXIMAL RISK CRITERIA

Consider the solution to the problem of calculating weight coefficients of importance $w \in D^4_w$ (6) using the generalized logical criterion "maximum risk":

$$x^* = \arg \max_{w \in D_w} \max_{1 \leqslant i \leqslant n} F(q(x), w) \tag{24}$$

subject to

$$D^1_w = \left\{ w \in R^n \;\middle|\; \begin{array}{l} w_i \geqslant w_0 \geqslant 0, \ i = 1, 2, \ldots, n; \\ \sum\limits_{i=1}^{n} w_i = R; \\ \langle w_i \geqslant g_l w_j \rangle_{e_l}, \ l = 1, 2, \ldots, L \end{array} \right\}. \tag{25}$$

For fixed values of the variable parameters $x \in D$, the values of the vector criterion $q(x)$ are definite as well.

The optimal solution of the optimization task (24)–(25) is the w^*, which has components:

$$w^*_i = \begin{cases} \dfrac{(R' \bar{g}^r_i)}{\sum\limits_{k \in V_r} \bar{g}^r_k} + \bar{\bar{g}}_i w_0, & i \in V_r; \\ \bar{\bar{g}}_i w_0, & i \notin V_r, \end{cases} \tag{26}$$

here

$$R' = R - w_0 \times \sum_{i=1}^{n} \bar{\bar{g}}_i \geqslant 0, \tag{27}$$

values $\bar{g}^r_i$ and $\bar{\bar{g}}_i$ are determined by (16) and (17), respectively; V_i is the set of vertices of the graph $G(V, E)$ from which there is a path to vertex i, including itself; n_i is the cardinality of the set V_i, value r is obtained from the relations:

$$y_r = \max_{1 \leqslant k \leqslant n} y_k, \tag{28}$$

$$y_k = \max_{i \in V_k} \left(q_i(x) \left(\frac{(R' \bar{g}^r_i)}{\sum\limits_{j \in V_r} \bar{g}^r_j} + \bar{\bar{g}}_i w_0 \right) \right). \tag{29}$$

The proof of formulas (24)–(29) is given in [10].

7. CONCLUSION

A technique for calculating the weight coefficients of the importance of particular criteria in solving problems of multi-criteria choice and evaluation is considered. The fundamental principle of different values of importance weight coefficients with a constant system of qualitative preferences allows the decision maker to formulate preferences in the form of a directed graph, which, in the case of completeness of the graph, is a strict order relation. This technique can be used both in building interactive decision-making systems and for reducing a multi-criteria problem into a single-criteria one for use in various optimization methods.

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